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The aging relation for Ising spin glasses

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Abstract. We present a rigorous dynamical relation for aging phenomena—the aging relation—for Ising spin glasses using the method of gauge transformation. The waiting-time dependence of the autocorrelation function in the zero-field-cooling process is equivalent to that in the field-quenching process. There is no aging on the Nishimori line; this provides arguments for dynamical properties of the Griffiths phase and the mixed phase. The present method can be applied to other complex systems with gauge symmetry such as the XY gauge glass.

Slow dynamics is an important concept in the study of complex systems, such as spin glasses, structure glasses, polymers, superconductors, and neural networks. It is one of the peculiar properties characterizing the spin-glass (SG) phase [1–3]. The aging phenomenon [4] is a typical realization of slow dynamics, especially for the SG [5–8]. It was first observed for a metallic SG material, CuMn, in a zero-field-cooling (ZFC) process [5]. The relaxation of the isothermal remanent magnetization depends on the waiting time for which the sample is kept at constant temperature prior to the application of the field. A similar waiting-time dependence was observed for other metallic SG materials and short-range SG materials (e.g. AgMn, AuFe, $Fe_xMn_{1-x}TiO_3$, and $CdCr_xIn_{1-x}S_4$), and in measurements for a field-cooling (FC) process [7, 8]. Experiments dictate that the aging in the ZFC process is provided by the removal of a sufficiently strong magnetic field applied to SG materials [6, 8]. This process is called field quenching (FQ), and is not equivalent to FC for which the applied field is weaker. While the FQ and the FC are different, the dynamical equivalence of ZFC and FQ is interesting if one contrasts it with the irreversibility observed in the ZFC and the FC processes.

Several attempts have been made theoretically to explain the aging by means of phenomenological arguments [9–11]. Cugliandolo and Kurchan [12] investigated the aging for the Sherrington–Kirkpatrick (SK) model analytically. They examined the autocorrelation function for the non-equilibrium process starting from a random state (ZFC from $T = \infty$). Rieger [13] investigated the aging in the 3D $\pm J$ Ising model by Monte Carlo simulation. He measured the waiting-time dependence of the autocorrelation function from the all-up state (FQ; see below). In none of these studies has the equivalence of ZFC and FQ been discussed theoretically, beyond the phenomenological arguments.

Since randomness and frustration make it difficult to examine SG systems analytically as well as numerically, only few points have been definitely confirmed. The method of gauge transformation [14–17] is a powerful technique for deriving exact results for the $\pm J$ and the Gaussian Ising spin glasses irrespective of the dimensionality. It provides the internal

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energy and an upper bound on the specific heat as non-singular functions of the temperature on a special line in the randomness–temperature phase diagram. This line is called the Nishimori line [14]. Furthermore, it provides a plausible argument for the absence of re-entrant transitions from the FM phase to a non-FM one (the SG in 3d) [15]. The method has been generalized to other random systems with various symmetries such as the XY gauge glass [16]. Recently, it has been extended for treating dynamical systems [17]. Since the method is formalized quite systematically, once a new result is obtained for a specific system, it can be generalized to other gauge-symmetric systems.

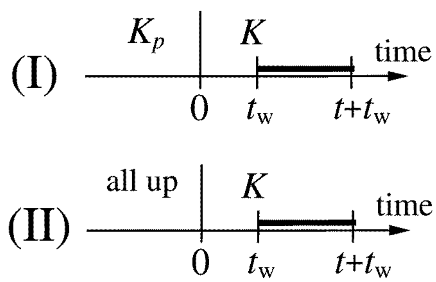

Figure 1. Illustrations of two non-equilibrium processes, I and II. The correlation is measured between $t_{\rm w}$ and $t + t_{\rm w}$.

In the present article, we derive a rigorous dynamical relation between two non-equilibrium processes, which are shown in figure 1, relating to aging phenomena for Ising SG models. The autocorrelation functions with a waiting time $t_{\rm w}$ for these processes satisfy

$$\left[\langle S_i(t_{\rm w})S_i(t+t_{\rm w})\rangle_K^{\boldsymbol{F}}\right]_{\rm c} = \left[\langle S_i(t_{\rm w})S_i(t+t_{\rm w})\rangle_K^{K_p}\right]_{\rm c} \tag{1}$$

where $K = J/k_{\rm B}T$ is the inverse temperature of the heat bath and K_p gives the effective (inverse) temperature characterizing the randomness (see later). Process I is related to the ZFC. At initial time $t = 0$, the system is kept in the equilibrium state at a temperature K_p with zero field; hereafter, we call K and K_p 'temperatures' instead of 'inverse temperatures'. The temperature is immediately changed (usually quenched) and the system relaxes in a heat bath at another temperature K for $t > 0$. The average for dynamical ensembles in this process is denoted by $\langle\cdots\rangle_K^{K_p}$. Process II is related to the FQ [6, 13]. The system starts from the all-up state $\boldsymbol{F} = (+, +, \ldots, +)$ at $t = 0$ and relaxes in the same heat bath as in process I for $t > 0$; the average is denoted by $\langle\cdots\rangle_K^{\boldsymbol{F}}$. Since the all-up state provides the strong-field limit, this represents the process with the field quenched from ∞ to zero at $t = 0$. Note that the FQ is not equivalent to the FC in which the applied field is weaker and is quenched after the waiting time. In the following, we show the derivation of equation (1) which we call the *aging relation* using the method of gauge transformation, and discuss the physical meaning and the applicability to other complex systems.

The Hamiltonian that we consider is $\mathcal{H} = J\widetilde{\mathcal{H}}(\boldsymbol{S};\boldsymbol{\omega}) = -J\sum_{\langle ij\rangle}\omega_{ij}S_iS_j$, where S_i takes the values ± 1, and $\boldsymbol{S} = (S_1, S_2, \ldots, S_N)$ represents a configuration of all N spins. The set $\boldsymbol{\omega} = (\omega_{12}, \ldots)$ represents a configuration of all $N_{\rm B}$ bonds, and the summation is taken over all bonds; while we make no restrictions on the type or the dimension of the lattice, one may suppose usual nearest-neighbour interactions on the d-dimensional hypercubic lattice. For a particular bond configuration $\boldsymbol{\omega}$, the equilibrium distribution is defined by $\rho_{\rm eq}(\boldsymbol{S}; K, \boldsymbol{\omega}) = \exp\{-K\widetilde{\mathcal{H}}(\boldsymbol{S};\boldsymbol{\omega})\}/Z(K, \boldsymbol{\omega})$. The exchange interaction $J_{ij} = J\omega_{ij}$ is a

Table 1. A summary of the variables and functions appearing in the bond distributions. N_B is the number of bonds. p is the concentration of $+J$ bonds for the $\pm J$ distribution, and J_0 is the centre of the distribution for the Gaussian distribution with the variance of unity.

	$\pm J$	Gaussian
ω_{ij}	± 1	Any real values
K_p	$\frac{1}{2}\ln(p/(1-p))$	J_0
$Y(K_p)$	$(2\cosh K_p)^{N_\mathrm{B}}$	$\exp\left(\frac{1}{2}N_\mathrm{B}{K_p}^2\right)$
$D(\boldsymbol{\omega})$	1	$\exp\left(-\frac{1}{2}\sum_{\langle ij\rangle}{\omega_{ij}}^2\right)$

random variable. The average for bond randomness is denoted by $[\cdots]_\mathrm{c}$. The general form of the bond distribution in a gauge-symmetric model [16, 17] is expressed as

$$P(\boldsymbol{\omega}; K_p) = \frac{D(\boldsymbol{\omega})}{Y(K_p)} \exp\left\{-K_p\widetilde{\mathcal{H}}(\boldsymbol{F};\boldsymbol{\omega})\right\} \tag{2}$$

where $\boldsymbol{F} = (+, +, \ldots, +)$. The functions ω_{ij}, K_p, $D(\boldsymbol{\omega})$ and $Y(K_p)$ are summarized in table 1. We treat both the $\pm J$ and the Gaussian bond distributions. In both types of distribution, K_p controls the randomness; $K_p = 0$ and ∞ correspond to the most random case and the non-random case, respectively. The Nishimori line is located on $K = K_p$.

Since the Ising system has no intrinsic dynamics, we consider a Markov process for each bond realization: the state distribution obeys the master equation [18], whose solution is formally given by $\rho_t(\boldsymbol{S}) = \sum_{\boldsymbol{S}'}\langle \boldsymbol{S}|\mathrm{e}^{t\boldsymbol{\Gamma}}|\boldsymbol{S}'\rangle\,\rho_0(\boldsymbol{S}')$. The matrix element $\langle \boldsymbol{S}|\mathrm{e}^{t\boldsymbol{\Gamma}}|\boldsymbol{S}'\rangle$ plays the role of a Green's function for a time interval t. The matrix $\boldsymbol{\Gamma}$ is composed of non-negative off-diagonal elements, and satisfies the detailed balance and the conservation of the probability. We consider both the Metropolis *et al* [19] and the Glauber dynamics [20]. The detailed expressions for $\boldsymbol{\Gamma}$ for these two types of dynamics are given in reference [17].

The gauge transformations for functions of $\boldsymbol{S}$ and $\boldsymbol{\omega}$ are defined by

$$U_{\boldsymbol{\sigma}}\colon S_i \longrightarrow S_i\sigma_i \tag{3}$$

$$V_{\boldsymbol{\sigma}}\colon \omega_{ij} \longrightarrow \omega_{ij}\sigma_i\sigma_j \tag{4}$$

where $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \ldots, \sigma_N)$ is an arbitrary state of N Ising spins. The Hamiltonian is invariant under the transformation $U_{\boldsymbol{\sigma}}V_{\boldsymbol{\sigma}}$. Note that we use the terminology 'gauge invariant' only for functions of the set $\boldsymbol{\omega}$ invariant under $V_{\boldsymbol{\sigma}}$. Following references [16, 17], the bond average of a gauge-invariant function $Q(\boldsymbol{\omega})$ can be expressed as

$$\left[Q(\boldsymbol{\omega})\right]_\mathrm{c} = \sum_{\boldsymbol{\omega}} \frac{D(\boldsymbol{\omega})Z(K_p,\boldsymbol{\omega})}{2^N Y(K_p)}\, Q(\boldsymbol{\omega}). \tag{5}$$

To treat dynamical systems, another transformation for $\boldsymbol{S}'$ is necessary:

$$U'_{\boldsymbol{\sigma}}\colon S'_i \longrightarrow S'_i\sigma_i. \tag{6}$$

We have shown the invariance of the time evolution:

$$U_{\boldsymbol{\sigma}}U'_{\boldsymbol{\sigma}}V_{\boldsymbol{\sigma}}\langle \boldsymbol{S}|\mathrm{e}^{t\boldsymbol{\Gamma}}|\boldsymbol{S}'\rangle = \langle \boldsymbol{S}|\mathrm{e}^{t\boldsymbol{\Gamma}}|\boldsymbol{S}'\rangle \tag{7}$$

for the Metropolis and the Glauber dynamics [17]. This leads us to the relations obtained previously:

$$\left[\langle S_i(t)\rangle_K^{\boldsymbol{F}}\right]_\mathrm{c} = \left[\langle S_i(0)S_i(t)\rangle_K^{K_p}\right]_\mathrm{c}. \tag{8}$$

Equation (8) is a generalization of the fluctuation-dissipation theorem in the region far from equilibrium.

We examine the aging by analysing the waiting-time dependence of non-equilibrium autocorrelation functions [12, 13] for processes I and II instead of the remanent magnetization, as in experiments. Since the autocorrelation function is a fundamental quantity for representing the dynamical structure in the system, we believe that it detects a typical aging feature if one exists. In the language of the master equation, the dependences of the functions are expressed as

$$\langle S_i(t_{\mathrm{w}})S_i(t+t_{\mathrm{w}})\rangle_K^{\boldsymbol{F}} = \sum_{\boldsymbol{S}_1\boldsymbol{S}_2} S_{2i}\,\langle \boldsymbol{S}_2|\mathrm{e}^{t\boldsymbol{\Gamma}}|\boldsymbol{S}_1\rangle\, S_{1i}\,\langle \boldsymbol{S}_1|\mathrm{e}^{t_{\mathrm{w}}\boldsymbol{\Gamma}}|\boldsymbol{F}\rangle \tag{9}$$

$$\langle S_i(t_{\mathrm{w}})S_i(t+t_{\mathrm{w}})\rangle_K^{K_p} = \sum_{\boldsymbol{S}_0\boldsymbol{S}_1\boldsymbol{S}_2} S_{2i}\,\langle \boldsymbol{S}_2|\mathrm{e}^{t\boldsymbol{\Gamma}}|\boldsymbol{S}_1\rangle\, S_{1i}\,\langle \boldsymbol{S}_1|\mathrm{e}^{t_{\mathrm{w}}\boldsymbol{\Gamma}}|\boldsymbol{S}_0\rangle\, \rho_{\mathrm{eq}}(\boldsymbol{S}_0;\, K_p, \boldsymbol{\omega}). \tag{10}$$

Using the same technique as was introduced in [17], one can easily see that $\langle S_i(t_{\mathrm{w}})S_i(t+t_{\mathrm{w}})\rangle_K^{\boldsymbol{F}}$ is transformed as

$$V_{\boldsymbol{\sigma}}\langle S_i(t_{\mathrm{w}})S_i(t+t_{\mathrm{w}})\rangle_K^{\boldsymbol{F}} = \langle S_i(t_{\mathrm{w}})S_i(t+t_{\mathrm{w}})\rangle_K^{\boldsymbol{\sigma}} \tag{11}$$

while $\langle S_i(t_{\mathrm{w}})S_i(t+t_{\mathrm{w}})\rangle_K^{K_p}$ is gauge invariant; $\langle\cdots\rangle_K^{\boldsymbol{\sigma}}$ expresses the average for the process starting from a fixed state $\boldsymbol{\sigma}$. Note that the probability distribution (2) is transformed as

$$V_{\boldsymbol{\sigma}}P(\boldsymbol{\omega};\, K_p) = \frac{D(\boldsymbol{\omega})Z(K_p, \boldsymbol{\omega})}{Y(K_p)}\,\rho_{\mathrm{eq}}(\boldsymbol{\sigma};\, K_p, \boldsymbol{\omega}). \tag{12}$$

Using equations (5), (11) and (12), we derive the relation (1) for any waiting time t_{w}, any time interval t, any temperature K and any degree of randomness K_p. We call it the 'aging relation', since it relates aging phenomena in two distinct processes whatever waiting-time dependence is essential for the aging. Equation (8) is a special case ($t_{\mathrm{w}} = 0$) of it.

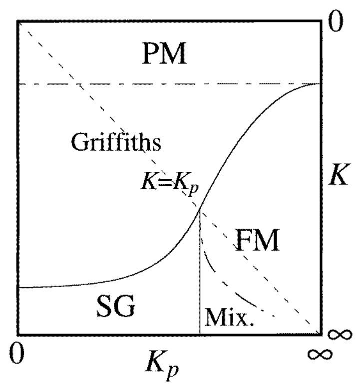

Figure 2. A typical phase diagram of Ising spin glasses in the K_p–K plane. The dashed line ($K = K_p$) is the Nishimori line. A possible Griffiths phase and mixed phase (Mix.) are indicated.

The aging relation contains parameters K and K_p characterizing the relaxation for $t > 0$. For each phase, one can examine the physical meaning of the relation by choosing (K, K_p) appropriately. A typical phase diagram in the randomness–temperature plane for Ising spin glasses is shown in figure 2; a possible Griffiths phase and mixed phase are indicated. Note that the initial temperature of process I is always located on $K = K_p$. In the SG phase where K_p indicates a high temperature (small K_p), process I is the ZFC process observed in experiments. Therefore, the aging relation relates aging phenomena for the ZFC and the FQ processes. The same relation was indicated by real experiments [6, 8], in which the waiting-time dependence of the remanent magnetization seemed equivalent in these two processes. Since the aging relation reveals just the equivalence of the gauge-invariant dynamical structure, the amplitude of the magnetization which is not gauge invariant does not obey such a relation, while the dynamical behaviour itself may obey it as observed in these experiments.

At a glance, the aging relation appears to express a trivial fact, since both initial states are located in the PM phase and both systems relax into the SG phase. However, the equivalence at any waiting time, which means the equivalence of the dynamical structure at any stage of relaxation, is non-trivial. It is interesting to compare this equivalence with the difference between the FC and the ZFC observed in experiments [1]. Further investigations in this direction would be helpful as regards achieving an understanding of the dynamical structure of the system.

Next, let us consider the aging relation on $K = K_p$ (the Nishimori line), where the temperature is kept constant at all times in process I. Then, the rhs of equation (1) becomes the equilibrium autocorrelation function, which is independent of the waiting time t_{w}:

$$\left[\langle S_i(t_{\mathrm{w}}) S_i(t + t_{\mathrm{w}})\rangle_K^F\right]_{\mathrm{c}} = \left[\langle S_i(0) S_i(t)\rangle_K^{\mathrm{eq}}\right]_{\mathrm{c}} . \qquad (13)$$

$\langle\cdots\rangle_K^{\mathrm{eq}}$ denotes the dynamical average for the equilibrium process. Equation (13) is derived from the fact that the equilibrium distribution $\rho_{\mathrm{eq}}(K)$ is an eigenstate of $\mathbf{\Gamma}$ with zero eigenvalue. Therefore, the autocorrelation function in the FQ process is independent of the waiting time, which suggests the absence of aging on the Nishimori line. Furthermore, equation (13) provides efficient Monte Carlo calculations for equilibrium autocorrelation functions; the equilibration can be omitted in the FQ process.

The aging has been considered an inherent property in the SG phase from experimental [6–8] and theoretical [9–11, 13] viewpoints. Since the aging phenomenon is a typical observation for the complex phase space for slow dynamics, it could occur in other complex phases. In such a phase, the aging would also be inherent, which means that it occurs throughout the whole of the phase when it is observed in some parts of the phase. It has been pointed out that there is a dynamically singular phase called the Griffiths phase [3, 21–24] between the critical temperature of the pure system and the phase boundary of the low-temperature phase (the FM or the SG)—see figure 2. However, the region of the Griffiths phase has not yet been determined definitely: figure 2 was proposed just from pursuing an analogy with the dilute ferromagnet [26]. Therefore, as regards the Griffiths phase, two cases can be considered to aid the understanding of the above result if the aging is an inherent property. (a) *There is no aging throughout the whole Griffiths phase*, if the Nishimori line intersects the Griffiths phase as in figure 2. (b) *There is no Griffiths phase at least around the Nishimori line.* In the former case, even if a slow dynamics is observed in the Griffiths phase, it is quite different from that in the SG phase [25]. The latter case allows the existence of the Griffiths phase below $K = K_p$. Another possibility is that (c) *the aging is not inherent at least in the Griffiths phase.* This means that the aging occurs in some parts of the Griffiths phase. Although it is not obvious which option is correct in the present framework, the above result restricts the existence of the aging phenomenon and the region of the Griffiths phase, and indicates that future investigations of them would be fruitful.

Following the results for the SK model and from experiments [1], one should consider the possibility of a mixed phase appearing between the FM and SG phases—see figure 2. It is reasonable to consider that the aging occurs in a mixed phase, since the SG feature in the mixed phase provides a typical slow dynamics which reveals the aging. Let us consider the temperature below the multicritical point on $K = K_p$, where the spontaneous magnetization appears. In such a region, it is not clear that the FQ process is appropriate for the observation of the aging, since the initial all-up state is not so different from the final equilibrium state with broken symmetry. As seen above, the amplitude of the magnetization is not important in the aging phenomenon. While the up–down symmetry is automatically broken in the FQ process, the dynamical behaviour is equivalent to that in the equilibrium (symmetric) process—see equation (13). Thus, we assume that the FQ process exhibits an aging feature even in such a symmetry-broken region. If the aging is inherent in the mixed phase, the relation (13) indicates that the Nishimori line does not enter the mixed phase. This restricts the topology of the phase diagram, and is consistent with the results from the SK model.

In summary, we have derived relation (1) for two non-equilibrium processes (the aging relation). The aging phenomenon in the zero-field-cooling process is equivalent to that in the field-quenching process. There is no aging on the Nishimori line. This provides some restrictions on the Griffiths and the mixed phases. Furthermore, efficient Monte Carlo calculations are possible for the equilibrium autocorrelation function on the Nishimori line. Similar relations can be derived for other gauge-invariant quantities such as the SG susceptibility. It can be generalized to other processes like the simulated annealing. The present theory is applicable to various systems: any gauge-symmetric distribution of the randomness instead of the Gaussian and the $\pm J$ distributions, any dynamics which satisfies equation (7) instead of the Glauber and the Metropolis ones, any dimensionality and lattice (e.g. the SK model). It can be extended to other gauge-symmetric systems such as the neural network system [27] and the XY gauge glass [16]; $\mathcal{H} = -J\sum_{\langle ij\rangle}\cos(\phi_i - \phi_j + A_{ij})$. It has been pointed out that granular systems of type-II superconductors in a magnetic field are described by this model. There is a similar relation for such materials; since no ordering field exists in superconducting systems, the all-up state is prepared from the ordered state in the pure system at sufficiently low temperature. In this literature, the parameter K_p controls the magnetic field. It is quite interesting that such a non-trivial relation holds in various complex systems irrespective of the details of the dynamics. Since unified treatments are generally difficult for complex systems, the present result could shed light on future investigations of them from the global viewpoint.

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